The task: Divide a square into X number of squares.

The subdivided parts must all be squares; but may be of different sizes.

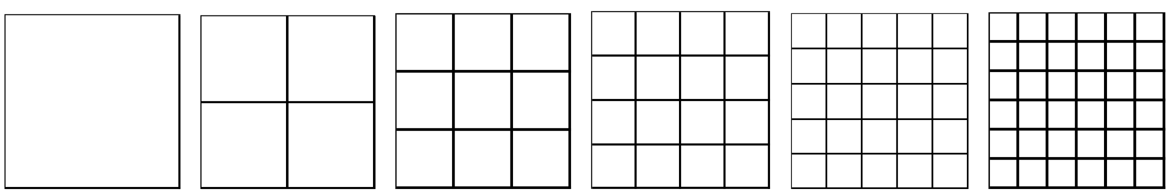
The subdivided squares must occupy the entirety of the original square.

The tricky number of squares usually asked is: 8. **8 total squares** within the overall square figure.

Rule 1: Only NXN matrixes can make squares

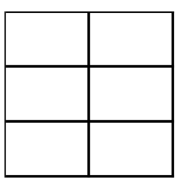
The dividing lines must fall on a matrix of N X N divisions, with N being any positive whole number. Total squares within the figure are given underneath the figure. For additional figures, larger squares can be consolidated out of smaller ones within the figure.

1X1 2X2 3X3 4X4 5X5 6X6



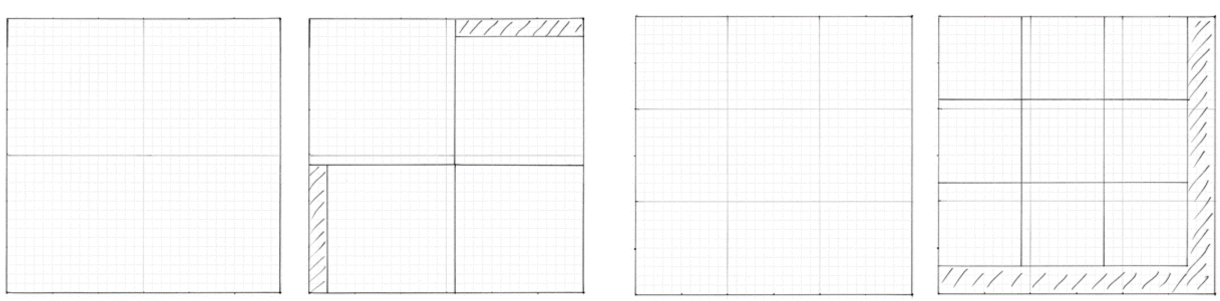
N=1 N=4 N=9 N=16 N=25 N=36

Any division by A X B (A and B being different whole numbers) will always result in rectangles, not squares. The simplest example is 2X3:



Think about this and see why this is true. It's true because squares are defined as having equal sides.

Here are a couple of examples of going slightly off of the "natural" NXN pattern. In every case that deviates from the NXN pattern, you are unable to fill the entire original square.



2X2 **Slightly off 2X2** **3X3** **Slightly off 3X3**

Rule 2: Subdivide and Consolidate

Squares in any figure can only be divided into whole number matrixes as shown in Rule 1, for the same reasons.

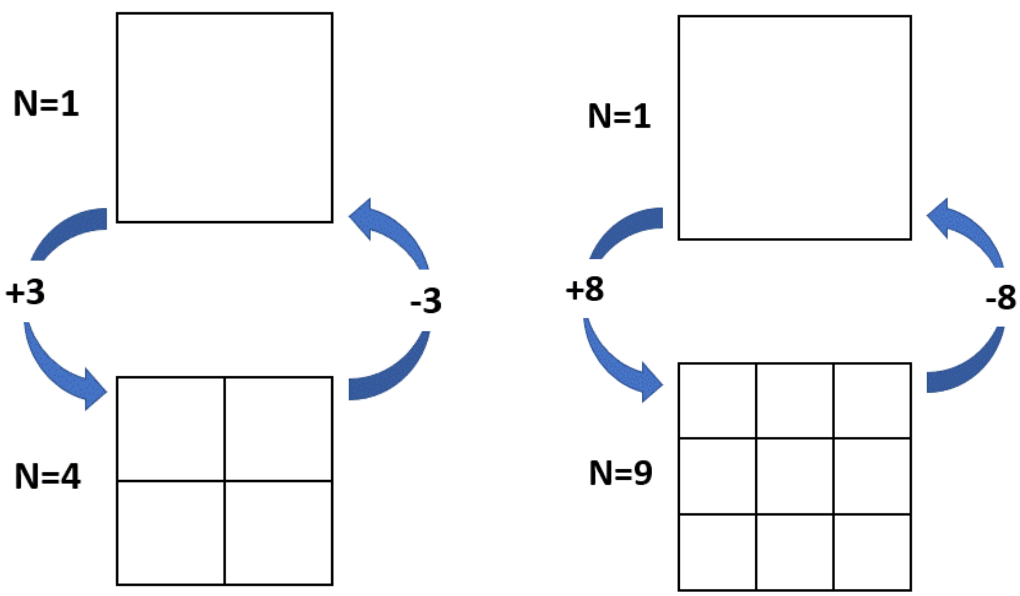
The simplest and most useful subdivisions are: 2X2 and 3X3. These turn out to all that are needed to produce every number of subdivided square segments, except the few that are not possible to make.

Dividing an existing square in a figure results in an addition of squares within the overall figure.

Consolidating an NXN figure within an existing pattern results in a subtraction of squares within the overall figure.

Dividing a square into a 2X2 matrix produces an additional 3 squares (the 4 resulting squares minus the one you had originally: 4 – 1 = +3). Consolidating a 2X2 matrix into a single square reduces the number of squares in the figure by 3 (1 – 4 = -3). So using 2X2 matrices, you can increment any figure higher by 3 squares (there's always a square available to subdivide) and reduce it by three, if there is already a 2X2 matrix within the figure.

Dividing a square into a 3X3 matrix produces an additional 8 squares (the 9 resulting squares minus the one you had originally: 9 – 1 = +8). Consolidating a 3X3 matrix into a single square reduces the number of squares in the figure by 8 (1 – 9 = -8). So using 3X3 matrices, you can increment any figure higher by 8 squares (there's always a square available to subdivide) and reduce it by eight, if there is already a 3X3 matrix within the figure. (For a 4X4 it's ± 15. For a 5X5 it's ± 24. For a 6X6 it's ± 35. Etc.)



There is no limit to how small one can subdivide the squares. (In math, with imaginary, infinitely narrow lines, the squares can be subdivided to an infinite level.)

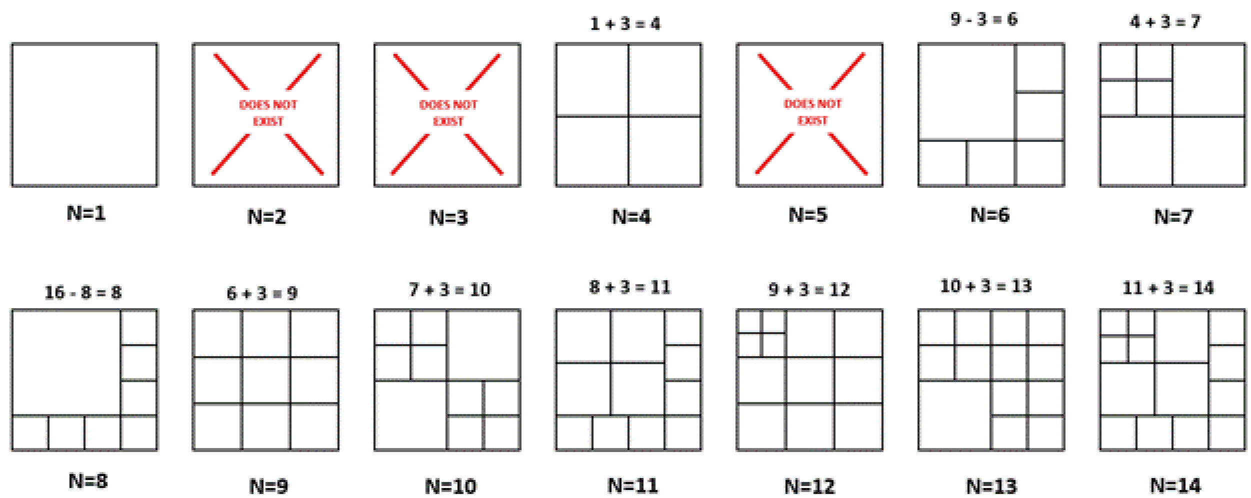
What counts of squares can be created?

All counts can be created except 2, 3, and 5. Since we can only increment our counts by +3, -3, +8, or -8 (and higher numbers), there's no way to get from 1 (or 4) to 2, 3, or 5. Since 5 doesn't exist, you can't consolidate back to 2. You also can't consolidate back to 3 from 6, because 6 has no 2X2 matrix in it to consolidate. (6 is created by consolidating a 2X2 within the figure having 9 squares. Larger squares have more than 5 residual squares, no matter what size square within the figure you consolidate.)

Once you have the first three numbers of squares in a row (e.g. 6, 7, and 8), then you can create all other higher numbers by incrementing +3 by repeatedly subdividing existing squares 2X2 in the prior figures:

9 = 6 + 3
10 = 7 + 3
11 = 8 + 3, etc.

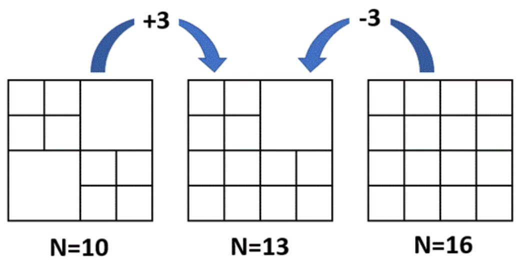
The first 3 counts in a row are figures with: 6, 7, and 8 squares each. Therefore, every number of squares 6 or higher can be created. Here is an example series. For some of the low numbers, only one pattern works, e.g. 1, 4, and 6.



Why is N=8 hard to wrap one's head around? No one naturally "wants" to think up to 16 and then back down to 8. So it's more tricky. Once you see the rules and the pattern, though, it's obvious.

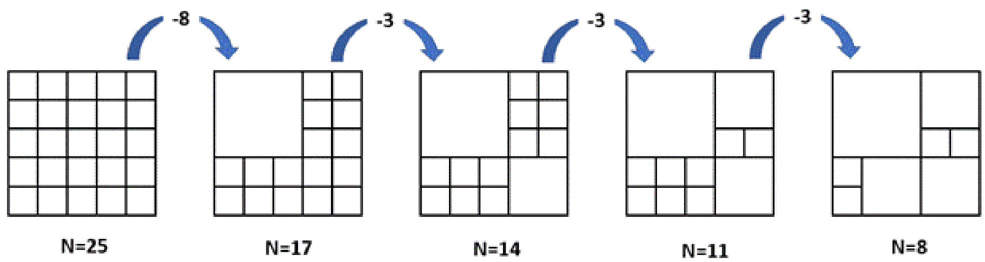
Without the rules we established, higher numbers become hard to intuitively imagine.

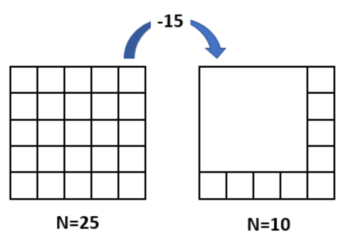
Once you reach higher numbers, more option present themselves. Also, number can be created from two directions, for instance, 13 can be viewed as either 10+3 or as 16-3:



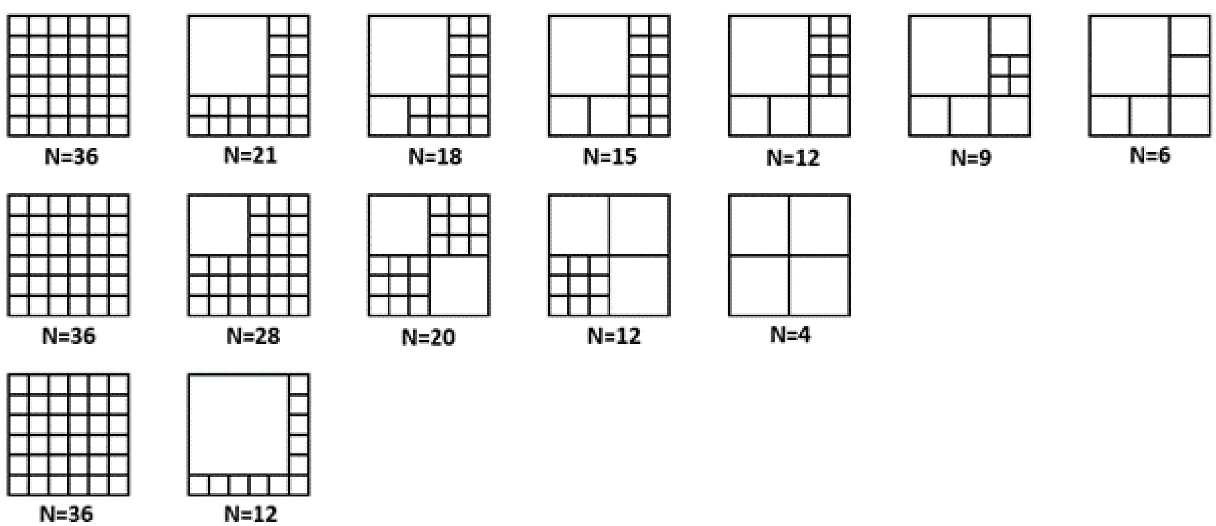
Note that odd configuration of the N=14 figure, above. Once you get to larger base matrixes, 5X5 and higher, there are more options on how to form various counts. See below.

This shows a less intuitive way to create counts of: 14, 11, 10, and 8, starting from a 5X5. But using the rules, it's completely simple.



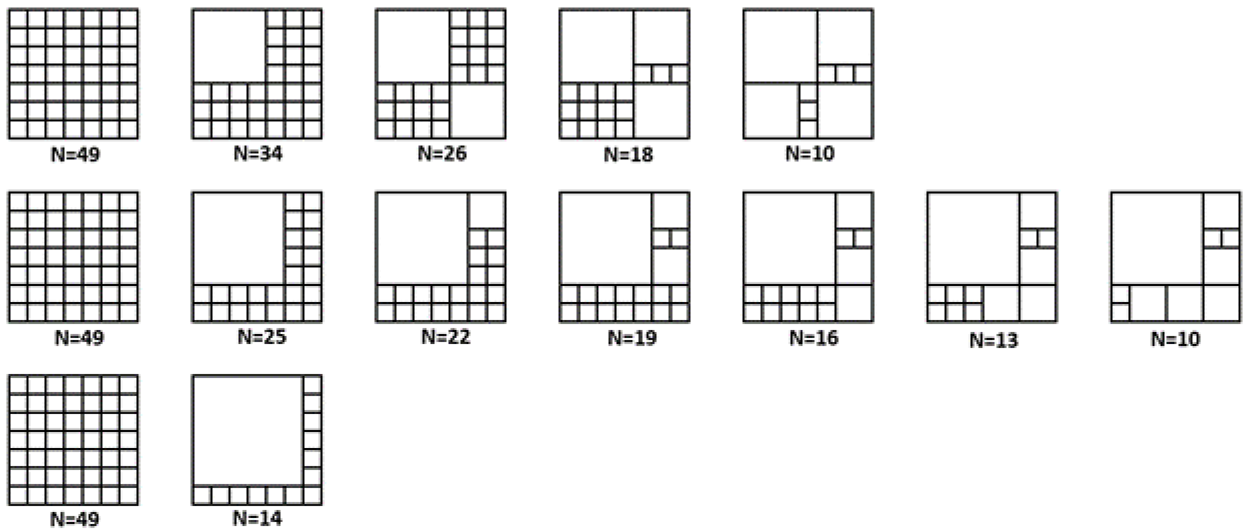


This figures shows the (maximum) reducing permutations reducing from a 6X6:



The reducing maximizes when you always choose the largest subfigures to consolidate. But depending on the relative sizes, you end up at different sized minimum counts. But never at 5, 3, or 2.

This figure shows reducing permutations from a 7X7:



You can see that going with larger numbers of starting squares does not get you closer to 5. If you can't get to 5 from 8, you cannot get there.

No matter how you adjust the large (or corner) square in the figure of 4, 6, or 8, you can't get just 4 smaller squares (the number of smaller squares has to be odd due to the corner).